Abstract

The post processing methods for interpreting flow characteristics from the annular cascade at the KTH Division of Heat and Power Technology are replaced with those used by a test turbine in order to provide more reliable results at high Mach numbers. The resulting measurements are compared against previous studies conducted at KTH in order to determine the differences in flow angle, speed, and pressure calculations. The new process results in an increase in yaw values of about 1° and smoother pressure distributions at high Mach numbers. The technique for multidimensional matrix construction and post processing application are described here in order to be reproduced and used in future studies. Due to the highly interpolated nature of the new method, varying matrix construction methods were tested, showing cubic interpolation to yield results with a further increase in exit yaw angle.
# ABSTRACT

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1 INTRODUCTION

1.1 Background
The Royal Institute of Technology’s Division of Heat and Power Technology utilizes three wind tunnels for research purposes. One is a specialized annular cascade rig with three turbine stators used for aerodynamic loss measurements. Another is a semi open calibration rig, and due to limitations, the probe calibrations can only be carried out at speeds up to Mach 0.8. Since the annular cascade can see supersonic speeds in testing, calibration data from the probe manufacturer, Aeroprobe, and from KTH must be combined to cover the full range of Mach numbers that the rig will experience. The third rig is a complete test turbine. The cascade rig and test turbines utilize different post processing methods to determine the total pressure, speed, and direction of flow.

1.2 Purpose
With the limitations of in-house probe calibration as mentioned previously, evaluations at higher Mach numbers are interpolated from the data provided by Aeroprobe. Turbines are continually pushed to their limits in order to produce maximum efficiencies and power. Therefore, it is important to determine the characteristics of the flow for all operating points. This work is a supplement to László Lilienberg’s (2016) thesis on experimental loss calculations and is done to clarify the results for high Mach number flow.

This report establishes and implements a new set of calibration coefficient equations for supersonic air speeds in the cascade rig. The new coefficients are based on the processing methods used by the test turbine, which are believed to be more reliable than the current methods used for the cascade. Variations with the new method are tested and analyzed to determine the best form for future use. A one-off test of the probe and calibration rig is done to validate prior data and determine the need for recalibration.
2 FRAME OF REFERENCE

2.1 Prior Work
The probe calibration is based on work done by Pradeep Srinivasan and Dr. John McClean. Mr. Srinivasan (2015) built the calibration matrices for use with the previous methods shown in Table 1, and in doing so converted the Aeroprobe data into the same form as the output from the KTH calibration tests. The interpolated data was used to from the new coefficient matrix based on the method used by Dr. McClean.

The method was verified using results obtained by László (2016) in his study of experimental loss calculations. In his thesis, secondary flow for cooled and uncooled vanes is analyzed in an annular cascade in order to mimic real-world results as compared to linear cascades that were used previously. Experiments were run on the KTH annular cascade and the flows were visualized using an in-house MATLAB script. The plots used for comparisons in this report are the Mach number, yaw, and pressure measurements.

2.2 Definitions
The measurements and calculations done with the probe data are interpreted so that positive yaw is associated clockwise rotation when pointing upstream. Figure 1 illustrates this orientation.

![Figure 1. Probe Orientation.](image)

The differences in equations, shown in Table 1, are the change in denominator and the introduction of the Mach coefficient. Due to this new Mach coefficient, the dynamic pressure coefficient is no longer necessary to calculate the static pressure after processing. The coefficients maintain the same numerators, meaning that general observations regarding the shape of coefficient plots and behavior of the results should be consistent across methods.

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Previous</th>
<th>New</th>
</tr>
</thead>
<tbody>
<tr>
<td>(k_{\text{yar}})</td>
<td>(p_3 - p_2)</td>
<td>(p_0 - p_1)</td>
</tr>
<tr>
<td>(k_{\text{pitch}})</td>
<td>(\frac{1}{p_3} (p_2 p_3 + p_1 + p_3))</td>
<td>(\frac{1}{p_3} (p_2 p_3 + p_1 + p_3))</td>
</tr>
<tr>
<td>(k_{\text{total}})</td>
<td>(\frac{1}{p_3} (p_2 p_3 + p_1 + p_3))</td>
<td>(\frac{1}{p_3} (p_2 p_3 + p_1 + p_3))</td>
</tr>
<tr>
<td>(k_{\text{dynamic}})</td>
<td>(\frac{1}{p_3} (p_2 p_3 + p_1 + p_3))</td>
<td>(\frac{1}{p_3} (p_2 p_3 + p_1 + p_3))</td>
</tr>
<tr>
<td>(k_{\text{Mach}})</td>
<td>(\frac{2}{\sqrt{\frac{k+1}{2}} (p_1 - p_2) + (p_1 - p_2)^2 + (p_1 - p_2)^2 + (p_1 - p_2)^2})</td>
<td>(\frac{2}{\sqrt{\frac{k+1}{2}} (p_1 - p_2) + (p_1 - p_2)^2 + (p_1 - p_2)^2 + (p_1 - p_2)^2})</td>
</tr>
</tbody>
</table>

Table 1. Calibration Coefficient Equations
With the probe orientation and equations, an increase in yaw angle would result in $p_5$ becoming larger than $p_4$, and therefore a decrease in $k_{yaw}$. In order to achieve this across all Mach numbers, the calibration data from Aeroprobe had to be modified by inverting the yaw values. This is discussed further in Section 3.1.

### 2.3 Evaluation Methods

The original KTH post processing script relies on an iterative method for determining the desired pressures and Mach number from the data. In this method, $k_{yaw}$ and $k_{pitch}$ are calculated and used with an approximated Mach number to find the nearest yaw and pitch angles. The other coefficients, $k_{static}$ and $k_{total}$ are then calculated based on these angles and used to compute $p_{total}$ and $p_{static}$, which are then used to calculate a new apparent Mach number. The process is repeated until the solution converges. The method is visualized below in Figure 2.

![Figure 2. KTH Iterative Method (Fridh, 2010).](image)

Dr. McClean’s method involves first computing the $k_{yaw}$, $k_{pitch}$, and $k_{Mach}$ for all of the data points. These values are then interpolated in a calibration matrix at a given level for a certain variable, yielding yaw, pitch, Mach, and $k_{total}$ values. The total pressure is calculated using $k_{total}$ and the static pressure is calculated by reversing the Mach equation used in Figure 2. The equations are reproduced below.

$$ p_0 = k_{tot} \cdot \frac{1}{2\sqrt{2}} (p_1 - p_2)^2 + (p_1 - p_3)^2 + (p_1 - p_4)^2 + (p_1 - p_5)^2 + p_1 \quad (1) $$

$$ p_s = \frac{p_0}{\left( M^2 \left( \frac{\gamma - 1}{2} \right) + 1 \right)^{\frac{\gamma}{2}}} \quad (2) $$

The calibration matrix from Dr. McClean’s method is an inverted form of the original that relies on interpolating the parameters from the coefficient values. This means that the interpolation and extrapolation methods used to construct the matrix, such as linear or cubic, impact the results. Their differences are discussed in a later section.
3 THE PROCESS

3.1 Matrix Construction

In order to implement the new coefficients, the coefficient matrix needed to be recalculated and reformed. Dr. McClean’s method took a form of the matrix used in the KTH method, but used an additional step of creating an application matrix for the coefficients so that they could be used with gridded interpolation in MATLAB.

First, the KTH matrix was recalculated using the new coefficient definitions given in Table 1. A test was performed in order to verify the application of the definitions, and the results are shown in Figure 3. The sample for a speed of Mach 1.10 shows that as the yaw angle increases at a constant pitch, \( k_{\text{yaw}} \) decreases, thus verifying the matrix construction method and probe orientation. In order for this to be the case across all Mach numbers, the yaw values in the Aeroprobe data needed to be inverted, as their definition of yaw was different than defined previously.

A new multi-dimensional matrix was then created according to Dr. McClean’s method. Through three stages of interpolation, the matrix (3) is formed so that the x, y, and z coordinates are the \( k_{\text{yaw}}, k_{\text{pitch}}, \) and \( k_{\text{Mach}} \) values, respectively, with the 4th coordinate representing page values such as yaw or pitch angle, Mach number, or \( k_{\text{total}} \) value.

\[
\begin{bmatrix}
k_{\text{yaw}} \text{ value}, & k_{\text{pitch}} \text{ value}, & k_{\text{Mach}} \text{ value}, & \text{Yaw angle} \\
& & & \text{Pitch angle} \\
& & & \text{Mach value} \\
& & & k_{\text{yaw}} \\
& & & k_{\text{pitch}} \\
& & & k_{\text{Mach}} \\
& & & k_{\text{total}} 
\end{bmatrix}
\]
This matrix was constructed using four different methods: linear interpolation with 100 coordinate coefficient values, linear with 50 coefficient values, linear with 200 coefficient values, and piecewise cubic interpolation using 100 values. The default is linear interpolation with 100 values, as those were the parameters used by Dr. McClean.

3.2 Post Processing

In order to apply the new coefficients, the post processing files needed to be updated. This mainly involved one evaluation file. Here, Dr. McClean’s method was inserted in place of the iterative method used previously. The changes follow the method described in Section 2.3, using MATLAB’s `griddedInterpolant` function to achieve the results. The function’s inputs are set for linear interpolation and nearest neighbor extrapolation. Previous calculations for uncertainty were left in the file in order to reduce other errors in the code, however their values are no longer associated with the plots. Re-implementing these calculations is recommended for further study. The new code can be seen in Appendix A.1. The changes fit within the scope of the previous post processing files used by Dr. Fridh for the cascade rig, so no major reconstruction of the overall system is necessary.
In this section, the results using the new method are compared with those produced in Lázsló’s report and among the different interpolation methods described in Section 3.

### 4.1 Flow Comparisons

The area averaged yaw plots in Figure 4 show an average mid-span flow angle of about 16.75° for the old method, and just over 17° for the new. The cascade has an absolute exit flow angle of 16.05°. The new method reaches a max of around 21.5°, whereas the old only reaches 20.5°. Both plots exhibit fairly consistent characteristics in terms of local deviations, particularly at mid-span. More yaw plots, found in Appendix A.2, show a similar ~1° increase at mid-span for the new method, as well as more extreme peaks around the hub walls.

![Old: Area averaged yaw, ASC38-106, $M_{iso} = 0.92989$](image1)

![New: Area averaged yaw, ASC38-106, $M_{iso} = 0.92989$](image2)

Figure 4. Mach 0.95 area averaged yaw results, old method on top.
There are no significant differences in the Mach number distribution plots shown in Figure 5. The new method seems to show slightly more detail in the low speed region of dark blue near the top of the plot. There are lighter regions at mid-span, however the scales are different and the values are similar.
For higher Mach numbers, the new method shows smoother transition areas, as demonstrated in the left side of Figure 6. The high velocity area mid-span is broken apart and is lighter colored than before. The scales are once again different, however the darker regions in the old plot represent larger values than the darkest regions in the new plot. Given an isentropic speed of Mach 1.05, the new smaller values seem more likely.
The total pressure distribution remains basically unchanged for lower Mach numbers, as there is less pressure change throughout the flow. However, as shown in Figure 7, higher speeds processed with the new method show fewer pressure deviations mid-span. This is also evident in the higher speed flow plots in the Appendix.

**4.2 Matrix Construction Variations**

The plots in Figures 8 and 9 show the results of varying the construction of the calibration matrix. The top three plots in Figure 8 display the results using grid sizes of 100, 50, and 200, respectively. There is no observable difference between the plots. When the calibration matrix was formed using piecewise cubic interpolation, however, the mid-span values increase by about half of a degree.
Figure 8. Mach 1.15 varying matrix construction.
Just as in the yaw plots, there is little deviation in the Mach number distribution for differing grid sizes, so they are not included here. Figure 8 shows the differences between results using linear versus piecewise cubic interpolation. They differ slightly, however both display the same general speed characteristics including the high and low speed regions and certain features near the hubs, like the lower speed zone in the bottom right corner as well as the arrow shape in the top left.
5 DISCUSSION AND CONCLUSIONS

5.1 Discussion

The plots in Section 4 and those included in the Appendix show the advantages and disadvantages of Dr. McClean’s method. The new method maintains the shape and speed characteristics of the vortices near the hub walls, which are expected given the historical knowledge and study of secondary flows. Any major deviation from the previous results would be unexpected. However, the new results do show slightly different yaw values and slightly more detail in low velocity areas. The yaw values are almost universally larger by 1° in the new plots. The flow angle continues to increase about 0.5 for each increase in Mach number, as concluded in László’s (2016) report. Around the hub walls, the new method shows more extreme yaw values, evidence of increased turning in the vortices.

Dr. McClean’s method displays more uniform pressure distribution at mid-span for high Mach numbers. This is evident in Figure 7 and echoed in the Appendix. Whereas previous results showed pockets of variable pressures at mid-span, the new method shows much less variation and is more representative of the expected flow through the vanes. The Mach number distribution plots, shown in Figure 6, show the opposite of the uniform pressure distribution plots. More uniform zones in plots of the old method give way to more varied regions with the new method.

The format of the calibration matrix only impacts the data when the interpolation method is changed. Differing grid sizes have no effect on the results. In the interest of matrix size and computational time, 100 points is recommended as the default. The further increase in yaw values with PCHIP interpolation, suggest that the interpolation is effecting the results. This change, coupled with the established use of linear interpolation in Dr. McClean’s code, suggest maintaining the linear construction method, although the method of choice is left to the experimenter.

5.2 Conclusions

The revised calibration coefficients show promising results for interpreting high Mach number data in the cascade rig. There are no significant deviations from prior results, which validates the method and procedures outlined by this report. The method is more stable for pressure readings at high Mach numbers which is important for loss calculations. The slightly higher yaw angles for high velocity flow can also be used as an important design consideration given that minor improvements in efficiency are important in turbomachinery. Final approval and implementation is left to Dr. Fridh. Work still needs to be done on the errors mentioned previously, however, if the results with uncertainties are to be used in future experimentation.

The code used and modified for the purposes of this report is passed along to Dr. Fridh for future implementation in the cascade rig.
6 RECOMMENDATIONS AND FUTURE WORK

Initially it was decided to run the calibration rig in order to acquire new probe data. The probe was found to be deformed and upon realigning the tip, one test at Mach 0.5 was run. The results showed deviations in pressure readings, as shown for $k_{yaw}$ in Table 2, so it is recommended to re-calibrate the probe before any further experimentation.

Table 2. Yaw Coefficient Differences

<table>
<thead>
<tr>
<th>$k_{yaw}$</th>
<th>2017</th>
<th>2015</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max</td>
<td>+1.828569</td>
<td>+1.926615</td>
</tr>
<tr>
<td>Mean</td>
<td>-0.061185</td>
<td>+0.024943</td>
</tr>
<tr>
<td>Min</td>
<td>-1.742043</td>
<td>-1.589599</td>
</tr>
<tr>
<td>Max Difference</td>
<td>0.177401</td>
<td></td>
</tr>
<tr>
<td>Mean Difference</td>
<td>0.086564</td>
<td></td>
</tr>
<tr>
<td>Min Difference</td>
<td>0.003521</td>
<td></td>
</tr>
</tbody>
</table>

The tests were stopped, however, as the László’s own calibration data was used to maintain consistency in calculations.

Future work could study the methods of interpolation and extrapolation using statistical or experimental methods. A test section could be run using two measurement grid sizes and the interpolation methods of the sparser grid could be compared to the values of the finer grid in order to determine the most accurate method.

Finally, the knowledge of an increase in yaw values for the main flow could be tested using computational or experimental methods to determine if a change in blade angles would be advantageous for turbine efficiency.
Fridh, Jens (2010), *Calibration Trials of Rapid Prototyping Probes* (EKV 05/10). KTH Royal Institute of Technology.


A.1 eval_5hole_20170718.m

```matlab
function
[p0,ps,M,al,be,wp0,wps,wM]=eval_5hole_20170718(p1,p2,p3,p4,p5,path_calib,fname_calib,\ngamma,n_loop,n_iter,rep_opt,Wp1,Wp2,Wp3,Wp4,Wp5,a_gamma)

% function for evaluating 5-hole aerodynamic probe data
% yielding yaw, pitch, p0, ps and Mach
% new algorithm with gridded interpolation, based on methods of John
% McClean
% Calculates experimental coefficient values, then plugs values into
% calibration matrix grid to find al, be, kTot, and Mach values.
% Jack Siman
% KTH/HPT
% 20170718

%% Data Loading and Preparation
% loading calibration data
load([path_calib fname_calib])
% app - matrix of calibration data

%----------------------------------------
% calculate coefficient values for data
kyaw = (p4-p5)./den;
kPitch = (p2-p3)./den;
kMach = sqrt(2./(gamma-1).*(p1./(p1-den)).^((gamma-1)./gamma)-1));

% OBSOLETE: for error calculations, but kept to avoid error
p_ave=(p2+p3+p4+p5)/4;
wp_ave=sqrt((0.25*Wp2).^2+(0.25*Wp3).^2+(0.25*Wp4).^2+(0.25*Wp5).^2);

%----------------------------------------
% extracting data from calibration matrix
X1 = squeeze(app(:,:,:,:4)); % kYaw coordinates
X2 = squeeze(app(:,:,:,:5));
X3 = squeeze(app(:,:,:,:6)); % experimental data
Xq1 = kYaw;
Xq2 = kPitch;
Xq3 = kMach;

%% Interpolation
% type of interpolation and extrapolation
type = 'cubic';
extrap_type = 'nearest';
% Yaw Values
V = squeeze(app(:,:,1,1));
Interp = griddedInterpolant(X1,X2,X3,V,type,extrap_type);
al = Interp(Xq1,Xq2,Xq3);

% Pitch Values
V = squeeze(app(:,:,2,1));
Interp = griddedInterpolant(X1,X2,X3,V,type,extrap_type);
be = Interp(Xq1,Xq2,Xq3);

% Mach Values
V = squeeze(app(:,:,3,1));
Interp = griddedInterpolant(X1,X2,X3,V,type,extrap_type);
M = Interp(Xq1,Xq2,Xq3);

% kTot Values
V = squeeze(app(:,:,7,1));
Interp = griddedInterpolant(X1,X2,X3,V,type,extrap_type);
k1 = Interp(Xq1,Xq2,Xq3);

%% Determine remaining flow parameters
```

APPENDIX A: SUPPLEMENTARY INFORMATION
\% total pressure
p0=k1.*den+p1;

\%Reconstructed static pressure
ps = p0.\/((M.\^\text{2}.\*((\gamma-1)/2)+1).\*\gamma/(\gamma-1));

\% OBSOLETE: for error calculations, but kept to avoid error
dp0dp1=k1+1; dp0dp_ave=-k1;
wp0=sqrt((dp0dp1.*Wp1).\^\text{2}+(dp0dp_ave.*wp_ave).\^\text{2});

dpsdp1=0; dpsdp0=1.\/((M.\^\text{2}.\*((\gamma-1)/2)+1).\*\gamma/(\gamma-1)); dpsdp_ave=0;
wp0=sqrt((dpsdp1.*Wp1).\^\text{2}+(dpsdp0.*wp0).\^\text{2}+(dpsdp_ave.*wp_ave).\^\text{2});

dMdp0=sqrt(2./((\gamma-1)).\*sqrt((p0./ps).\*\gamma*(p0.\*\gamma/(\gamma-1)).\*\gamma*(p0.\*\gamma/(\gamma-1)).\*\gamma));
dMdp0=sqrt(2./((\gamma-1)).\*sqrt((p0./ps).\*\gamma*(p0.\*\gamma/(\gamma-1)).\*\gamma*(p0.\*\gamma/(\gamma-1)).\*\gamma));
wM=real(sqrt((dMdp0.*wp0).\^\text{2}+(dMdp0.*wp0).\^\text{2}));
end
A.2 Method Difference Plots

Old: Total pressure distribution, ASC38, ASC38-106 $M_{iso} = 0.93105$

New: Total pressure distribution, ASC38, ASC38-106 $M_{iso} = 0.93105$
Old: Total pressure distribution, ASC38, ASC38-107 \( M_{iso} = 1.0376 \)

New: Total pressure distribution, ASC38, ASC38-107 \( M_{iso} = 1.0376 \)
PCHIP: Total pressure distribution, ASC38, ASC38-109 \( M_{iso3} = NaN \)
Old: Mach number distribution, ASC38, ASC38-110 $M_{iso3} = 1.2054$

New: Mach number distribution, ASC38, ASC38-110 $M_{iso3} = 1.2054$
Old: Mach number distribution, ASC38, ASC38-111 $M_{iso} = 0.93618$

New: Mach number distribution, ASC38, ASC38-111 $M_{iso} = 0.93618$
Old: Total pressure distribution, ASC38, ASC38-111 $M_{iso3} = 0.93618$

New: Total pressure distribution, ASC38, ASC38-111 $M_{iso3} = 0.93618$